

Mathematics Pre-Service Teachers' Reasoning in Solving Three-Dimensional Geometry Problems: The Role of Spatial Visualization, Orientation, and Relations

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ABSTRACT: Three-dimensional (3D) geometry requires learners to coordinate spatial representations, geometric relationships, measurements, and mathematical properties. For mathematics pre-service teachers, understanding how these processes are enacted is important because their own geometric reasoning may influence their capacity to support students' reasoning. This study aims to characterize the reasoning of mathematics pre-service teachers in solving 3D geometry problems, with particular attention to the roles of spatial visualization, spatial orientation, and spatial relations. A qualitative descriptive approach was employed. The participants were three third-semester mathematics education students who had completed a geometry course and were selected based on high performance in spatial visualization, spatial orientation, and spatial relations, respectively, as well as high performance on a 3D geometry problem-solving task. Data were collected through spatial ability tests, a 3D geometry problem-solving task, task-based interviews, and analysis of participants' written and graphical work. The data were analyzed through data condensation, data display, and conclusion drawing and verification, with the reasoning processes examined across Polya's problem-solving stages. The findings revealed that all three participants demonstrated four types of 3D geometric reasoning: 3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties. These reasoning types were evident across understanding, planning and executing, and checking stages and were found to be interrelated rather than independent. Differences among participants were most apparent during representation and spatial transformation tasks. Spatial visualization was particularly evident in mentally manipulating and transforming 3D representations, spatial orientation was associated with interpreting objects from different viewpoints, and spatial relations supported the coordination of relationships among components of 3D configurations. The findings suggest that spatial ability provides distinct but complementary cognitive resources for 3D geometric reasoning and that mathematics teacher education should integrate spatial reasoning activities with conceptual and procedural learning in 3D geometry.

KEYWORDS: Mathematics Pre-service Teachers, 3D Geometry, Spatial Visualization, Spatial Orientation, Spatial Relations

I. INTRODUCTION

Mathematical reasoning is a fundamental cognitive process in mathematics because it enables individuals to establish relationships, construct arguments, and draw logical conclusions from available information. Rather than merely applying rules or procedures, mathematical reasoning involves the processes through which learners interpret information, connect mathematical concepts, and justify the validity of their conclusions (Setyawan & Rahman, 2013; Kyllonen, 2020). Reasoning is therefore regarded as an essential component of mathematical activity and an important foundation for conceptual understanding and problem solving (Hjelte et al., 2020; Guerin et al., 2021). In mathematics education, reasoning is not only considered a learning outcome but also an integral part of the process of learning mathematics, because mathematical knowledge is constructed through logical relationships and justified conclusions (Thompson et al., 2012; Sidenvall et al., 2014; Jeannotte & Kieran, 2017).

The role of reasoning becomes particularly important in geometry, where learners are required to coordinate visual representations, geometric properties, spatial relationships, and mathematical concepts. Geometry involves the study of shapes, space, structures, measurements, and relationships among geometric objects, requiring learners to visualize objects, identify their properties, and reason about spatial relationships (Kuzle & Glasnovic, 2020; Fujita et al., 2020). These demands become more complex in three-dimensional (3D) geometry because learners must coordinate information represented from different perspectives and reason about objects that cannot always be directly perceived in their entirety. Consequently, solving 3D geometry problems requires more than knowledge of geometric formulas; it involves the ability to manipulate mental representations, organize spatial structures, identify geometric relationships, and apply mathematical properties appropriately.

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Pittalis and Christou (2010) conceptualized 3D geometric reasoning through four interrelated types of reasoning: 3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties. 3D object manipulation involves mentally transforming or interpreting three-dimensional objects and their representations. Spatial structuring concerns identifying and coordinating relationships among the components of a 3D configuration. Measurement involves reasoning about geometric quantities such as length, area, and volume, whereas conceptualization of mathematical properties involves using and connecting geometric properties to justify conclusions. These forms of reasoning provide a useful framework for examining how learners construct solutions to 3D geometry problems because they capture not only whether a solution is correct but also how geometric information is interpreted, organized, and transformed during problem solving.

Despite the importance of these forms of reasoning, 3D geometry remains challenging for many learners. Previous studies have reported difficulties in representing 3D objects through two-dimensional drawings, interpreting spatial relationships, visualizing cross-sections, understanding spatial structures, and applying geometric properties appropriately (Hock et al., 2015; Luneta, 2015; Pittalis & Christou, 2010; Sudirman et al., 2023). Learners may, for example, recognize individual geometric elements but fail to establish the spatial relationships among them. Such difficulties may result in inappropriate representations or incorrect conclusions even when learners possess relevant procedural knowledge. Research has also shown that learners' experiences in 3D geometry vary according to their ability to visualize, interpret, and reason about geometric objects and their representations (Jones et al., 2012; Noto et al., 2019; Chiphambo & Mtsi, 2021). These findings suggest that examining the reasoning process itself is important for understanding why learners succeed or experience difficulties when solving 3D geometry problems.

One cognitive factor closely associated with 3D geometric reasoning is spatial ability. Spatial ability refers to the capacity to generate, retain, retrieve, and transform visual-spatial representations (Maier, 1998; Uttal et al., 2013). It is a multidimensional construct that includes spatial visualization, spatial orientation, and spatial relations (Lohman, 1979; Maier, 1998). Spatial visualization refers to the ability to construct and mentally transform representations of objects, including mentally rotating, folding, unfolding, or manipulating objects. Spatial orientation concerns the ability to understand the position and orientation of an object when the observer's viewpoint changes. Spatial relations involve recognizing, comparing, and coordinating spatial relationships among parts of an object or among different objects (McGee, 1979; Lohman, 1979; Maier, 1998). These components provide different but complementary cognitive resources for reasoning about 3D geometric objects.

The relationship between spatial ability and geometric reasoning can be understood more clearly by considering the specific demands of 3D geometry. Spatial visualization may support the manipulation of 3D objects because learners need to construct and transform mental images when an object is rotated, unfolded, or viewed from another position. It may also support spatial structuring by enabling learners to coordinate the components of a complex geometric configuration (Pittalis et al., 2009; Battista, 2007; Atit et al., 2022). Spatial orientation, in contrast, becomes particularly relevant when learners must interpret an object from different viewpoints and maintain the consistency of its geometric relationships despite changes in perspective (Hegarty & Waller, 2005; Atit et al., 2022). Spatial relations may facilitate the identification and coordination of relationships among points, edges, faces, lengths, angles, and other geometric elements, thereby supporting measurement and conceptualization of mathematical properties (Battista, 2007; Mix & Cheng, 2012; Atit et al., 2022). Thus, the three components of spatial ability may contribute to different aspects of reasoning rather than functioning as a single undifferentiated cognitive capacity.

Previous research has provided evidence that spatial ability is associated with success in geometry and 3D geometric tasks (Pittalis et al., 2009; Fujita et al., 2020; Atit et al., 2022). However, much of this research has emphasized relationships between spatial ability and achievement or performance, while less attention has been given to how specific spatial abilities are manifested in learners' actual reasoning processes while solving 3D geometry problems. Knowing that a learner with stronger spatial ability performs better on a geometry task does not necessarily explain how visualization, orientation, or spatial relations are mobilized during problem solving. In particular, it remains important to examine how learners use these spatial resources when interpreting a problem, developing a solution strategy, manipulating representations, establishing spatial relationships, applying geometric properties, and verifying their conclusions. A process-oriented investigation is therefore needed to reveal the nature of reasoning underlying successful or unsuccessful 3D geometry problem solving.

This issue is particularly important for mathematics pre-service teachers. Pre-service teachers are prospective facilitators of students' mathematical thinking and are expected not only to possess adequate geometric knowledge but also to understand how geometric reasoning develops during problem solving. Their own experiences in solving geometry problems may influence how they later represent geometric concepts, select instructional strategies, interpret students' difficulties, and support students' reasoning (Gorgorió et al., 2013; Makamure & Jojo, 2021; Dos Santos, 2017; Barham, 2019). Research on pre-service mathematics teachers is therefore important because their competence in geometry encompasses not only conceptual knowledge but also the ability to reason about geometric objects and representations. Understanding their reasoning processes can provide insights into the cognitive resources that future teachers mobilize when confronting complex 3D geometry problems (Panaoura & Gagatsis, 2009; Aslan-Tutak & Adams, 2015; Unal et al., 2009; Mubarik et al., 2020).

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A qualitative examination is particularly appropriate for this purpose because reasoning is manifested through the sequence of actions, representations, explanations, and justifications produced during problem solving. Written solutions alone may not fully reveal why a pre-service teacher chooses a particular representation, how an object is mentally transformed, how relationships among geometric elements are established, or why a particular geometric property is considered relevant. Interviews and analysis of students' problem-solving processes can provide access to these aspects of reasoning and reveal the interaction between spatial thinking and mathematical reasoning. Such an approach can distinguish, for example, whether a learner's difficulty arises from manipulating a 3D representation, maintaining an object's orientation, identifying spatial relationships, measuring geometric quantities, or conceptualizing the mathematical properties underlying the solution.

Accordingly, this study focuses on the reasoning of mathematics pre-service teachers when solving 3D geometry problems, with particular attention to the roles of spatial visualization, spatial orientation, and spatial relations. The study examines how these three components are manifested in the four types of 3D geometric reasoning proposed by Pittalis and Christou (2010): 3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties. Rather than focusing solely on the correctness of the final answer, the study seeks to characterize the reasoning processes through which pre-service teachers understand, represent, transform, relate, and justify geometric information during problem solving.

The study therefore addresses the following research question: How do spatial visualization, spatial orientation, and spatial relations manifest in mathematics pre-service teachers' reasoning when solving three-dimensional geometry problems? By addressing this question, the study seeks to contribute a more detailed understanding of the cognitive processes underlying 3D geometric reasoning and to provide a qualitative account of how different components of spatial ability support different forms of mathematical reasoning. The findings may also inform the preparation of mathematics pre-service teachers by identifying the spatial and reasoning processes that need to be strengthened in learning and teaching 3D geometry.

II. METHOD

1. Participants

The participants in this study were students who had completed the geometry course, namely third-semester undergraduate students of the Mathematics Education Program, Class 2023C, at Universitas Negeri Surabaya, totaling 42 students. They were given a spatial visualization ability test, a spatial orientation ability test, a spatial relations ability test, and a 3D geometry problem-solving task. Subsequently, one student with high spatial visualization ability (SVS), one student with high spatial orientation ability (SOS), and one student with high spatial relations ability (SRS) were selected. All three participants completed the tasks comprehensively and obtained high-category scores on the 3D geometry problem-solving task.

2. Research instruments

Four types of instruments were developed to achieve the objectives of this study, namely a spatial visualization ability test, a spatial orientation ability test, a spatial relations ability test, and a 3D geometry problem-solving task. Before being used to collect research data, all four instruments underwent expert validation, readability testing, and empirical testing. The empirical testing of the research instruments included examining construct validity, level of difficulty, discriminating power, reliability, and distractor index (specifically for multiple choice items).

3. Procedure

The spatial visualization (SV), spatial orientation (SO), and spatial relations (SR) tests were administered to 42 selected participants. The results of these tests were then scored, and each participant's score was categorized into a high group ($70 \leq x \leq 100$) or a low group ($0 \leq x < 70$). Based on the results, 25 participants were classified as having high spatial visualization ability and 17 as low. For spatial orientation ability, 24 participants were categorized as high and 18 as low. For spatial relations ability, 19 participants were classified as high and 23 as low. Thus, there were 25 participants with high spatial visualization ability, 24 with high spatial orientation ability, and 19 with high spatial relations ability.

One week later, the same students completed the 3D Geometry Problem-Solving Task (TPMG3D) to obtain data on students' reasoning in solving problems. The results of the TPMG3D were then scored and categorized into high ($70 \leq x \leq 100$) and low ($0 \leq x < 70$) groups. The results showed that 6 participants were in the high category and 32 participants were in the low category. The number of participants who completed the TPMG3D was reduced because four students were absent during data collection.

Next, one participant with high spatial visualization ability, one with high spatial orientation ability, and one with high spatial relations ability were selected. The selected participants were those who had completed all four tests and obtained high scores on both the three spatial ability tests and the TPMG3D. The three selected participants were then given a task equivalent to the TPMG3D to validate the research data. Afterward, interviews were conducted with the three participants to obtain more valid and accurate data. The interviews used in this study were task-based interviews. Finally, the collected data were analyzed through the stages of data condensation, data display, and conclusion drawing/verification (Miles et al., 2014). The results of the data analysis

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regarding reasoning in solving 3D geometry problems for the three research subjects were then described based on Polya's problem-solving stages.

III. RESULT AND DISCUSSION

1. Overview of Pre-Service Teachers' 3D Geometric Reasoning

The qualitative analysis revealed that all three mathematics pre-service teachers demonstrated the four types of reasoning proposed by Pittalis and Christou (2010): **3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties**. These four types of reasoning appeared throughout the four problem-solving stages: understanding the problem, planning the solution, executing the plan, and checking the solution. The findings indicate that 3D geometric reasoning is not characterized by a single cognitive process; rather, different forms of reasoning are activated and coordinated according to the demands of the problem.

At the **understanding stage**, all three participants established logical relationships between stacked 3D objects, between unit cubes and larger solids, between different views of a 3D object, and between geometric quantities and the corresponding solids. Thus, understanding a 3D geometry problem involved more than identifying the information explicitly stated in the problem. The participants constructed a mental representation of the geometric configuration and identified relationships among its components. This finding is consistent with the view that problem understanding involves identifying relevant information, clarifying the meaning of the problem, constructing representations, and connecting the given information with prior knowledge (Kuzle, 2011; Rott et al., 2021; Olivares et al., 2020).

At the **planning stage**, the participants transformed their understanding into strategies. They determined how to represent front, top, and side views; how to construct nets; how to determine the number of unit cubes forming a solid; how to determine dimensions; and how to calculate surface area. This indicates that spatial reasoning was not limited to perceiving a geometric configuration but was also involved in anticipating and organizing subsequent solution procedures. Such planning requires the coordination of conceptual and procedural knowledge and the evaluation of alternative strategies (Kuzle, 2011; Rott et al., 2021; Wahab et al., 2024).

At the **execution stage**, the four reasoning types became particularly visible. Participants manipulated representations of 3D objects, organized unit cubes into larger structures, established relationships among dimensions, and applied geometric properties in calculations and drawings. The execution stage also revealed the most prominent differences among the three participants. Although all participants demonstrated the four types of reasoning, their ability to represent, construct, and transform 3D objects varied according to the spatial ability component in which they were categorized.

At the **checking stage**, all participants verified the correspondence between their representations and the original 3D objects, checked the consistency of unit-cube arrangements, examined dimensions and calculations, and evaluated whether the mathematical properties used were appropriate. This finding supports previous descriptions of checking as a metacognitive process involving evaluation of procedures, calculations, concepts, and the consistency of the final solution with the problem requirements (Oudman et al., 2022; Rott et al., 2021; Kuzle, 2011).

Overall, the results demonstrate that the four types of reasoning identified by Pittalis and Christou (2010) provide a useful framework for characterizing pre-service teachers' reasoning in 3D geometry. However, the findings also indicate that these forms of reasoning are **interrelated rather than independent**. A participant may simultaneously manipulate a 3D representation, organize its spatial structure, determine its dimensions, and apply relevant mathematical properties.

2. Reasoning of the Spatial Visualization Subject

The participant categorized as having high **spatial visualization (VS)** demonstrated strong performance across the four types of reasoning. During the understanding stage, VS established relationships between stacked 3D objects and between front, top, and side views and the corresponding 3D object. VS also recognized the relationship between unit cubes and larger cubes or rectangular prisms. These behaviours indicate that spatial visualization supported the construction and transformation of mental representations of 3D objects.

During planning, VS was able to determine strategies for representing different views of a 3D object, constructing nets, determining the number of unit cubes, determining dimensions, and calculating surface areas. This pattern suggests that visualization was used not merely to "see" the object but to anticipate its possible representations and transformations before carrying out the solution. In this sense, spatial visualization functioned as a cognitive resource for coordinating 3D object manipulation and spatial structuring. The strongest evidence emerged during the execution stage. VS correctly represented the front, top, and side views of the 3D object and constructed several possible nets. VS was also able to mentally determine the number of unit cubes contained in a larger solid and use this structure to determine different rectangular-prism configurations. In measurement tasks, VS related the number of unit cubes to edge length, dimensions, and surface area. VS also applied congruence when determining surface areas and constructing representations.

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These findings are consistent with Pittalis and Christou (2010), who associated 3D geometric reasoning with the manipulation and structuring of geometric objects. The findings also support the broader argument that spatial visualization is important when learners need to mentally transform, construct, or decompose geometric representations. In the present study, this role was particularly evident when VS moved between a 3D configuration and its two-dimensional representations.

Importantly, however, the contribution of visualization was not restricted to manipulation. VS also demonstrated measurement and conceptual reasoning. Thus, spatial visualization appears to provide a representational foundation upon which other forms of mathematical reasoning can operate. The ability to mentally organize the object allowed VS to subsequently determine dimensions, identify relevant surfaces, and apply geometric properties.

At the checking stage, VS verified the correspondence between the constructed representations and the original object, checked the consistency of unit-cube arrangements, and evaluated the correctness of dimensions, formulas, operations, and mathematical properties. This suggests that visualization continued to operate beyond the initial construction of the representation and supported the verification of geometric consistency.

3. Reasoning of the Spatial Orientation Subject

The participant categorized as having high **spatial orientation (OS)** also demonstrated all four types of reasoning. At the understanding stage, OS established relationships between stacked 3D objects and their front, top, and side views and connected unit-cube arrangements with larger 3D solids. OS also related dimensions and volume to the underlying spatial structure.

During planning, OS developed strategies for identifying surfaces visible from different viewpoints and for constructing nets. OS also planned strategies for determining unit-cube arrangements, dimensions, and surface areas. These behaviors suggest that spatial orientation supported the interpretation of a geometric object when its position or viewpoint changed.

The execution stage, however, revealed a distinctive characteristic of OS. Although OS correctly represented the front and side views of the object, the **top view was not represented correctly**. OS nevertheless succeeded in constructing different nets and solving several measurement and surface-area tasks. In addition, OS was able to determine the height of a regular square pyramid based on the heights of the cube and rectangular prism in the stacked configuration.

This finding is particularly important because it demonstrates that high performance in one spatial component does not guarantee uniformly successful performance across all spatial transformations. Spatial orientation appears to be especially relevant when a task requires maintaining an object's configuration while changing the observer's viewpoint. The difficulty encountered by OS in producing the correct top view suggests that orientation-related processing may become more demanding when multiple spatial relationships must be coordinated simultaneously.

Nevertheless, OS successfully applied geometric properties in calculating the surface areas of cubes, rectangular prisms, and regular square pyramids and used properties of 3D objects when constructing representations and nets. Thus, the difficulty in one representation task did not reflect a general deficiency in geometric conceptualization.

At the checking stage, OS was able to verify the correspondence of views and nets with the relevant geometric properties and to check measurements, formulas, operations, and solution procedures. This suggests that orientation was involved not only in constructing representations but also in evaluating their spatial consistency.

4. Reasoning of the Spatial Relations Subject

The participant categorized as having high **spatial relations (RS)** demonstrated a pattern similar to the other participants but showed particular strength in identifying relationships among spatial elements. During the understanding stage, RS established relationships between stacked 3D objects, different views, unit-cube arrangements, dimensions, and volume.

During planning, RS was able to formulate strategies for representing different views, constructing nets, determining the number of unit cubes, identifying dimensions, and calculating surface areas. This indicates that spatial relations supported the coordination of multiple geometric elements rather than focusing on isolated features of the object.

During execution, RS correctly represented the front, top, and side views and constructed four different nets of the 3D object. RS also mentally determined the number of unit cubes in a solid and used the spatial structure to identify possible rectangular-prism configurations. In measurement tasks, RS determined edge lengths, pyramid height, and surface areas by relating different geometric quantities.

However, RS showed some limitations in constructing 3D rectangular-prism representations from unit cubes. Although RS produced two representations, the drawings did not completely follow the appropriate drawing procedure. This finding suggests that recognizing relationships among spatial components and accurately externalizing those relationships in a conventional geometric drawing are related but distinguishable processes.

The RS participant nevertheless demonstrated strong measurement and conceptual reasoning. RS successfully connected the dimensions of stacked objects and applied congruence and other geometric properties to solve surface-area problems. This supports the interpretation that spatial relations can facilitate the coordination of multiple geometric quantities and structural relationships.

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At the checking stage, RS verified the consistency of the 3D representations, the correspondence between views and the original object, the number of unit cubes, and the correctness of measurements, formulas, operations, and mathematical properties.

5. Cross-Case Comparison: Similarities and Differences

A cross-case comparison showed that the three participants displayed **substantially similar reasoning patterns** across the four problem-solving stages. All participants demonstrated 3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties. They also engaged in understanding, planning, execution, and checking.

The main differences appeared during the **execution stage**, particularly in tasks requiring external representation and spatial construction. The VS and RS participants correctly represented the front, top, and side views, whereas the OS participant had difficulty representing the top view. In spatial structuring, VS produced all six required rectangular-prism representations, RS produced two, whereas OS did not produce any. In measurement, OS and RS successfully determined the height of the regular square pyramid from the given dimensions, whereas VS did not.

These differences are theoretically meaningful. They suggest that spatial visualization, orientation, and relations may not operate as interchangeable abilities. Instead, each may provide different cognitive resources for particular aspects of 3D geometric reasoning. Visualization appears particularly evident in mental construction and transformation; orientation becomes salient when viewpoints must be coordinated; and spatial relations become important when multiple geometric components and their relationships must be coordinated.

At the same time, the three participants showed **no substantial difference in conceptualization of mathematical properties**. All participants were able to identify, understand, and apply relevant geometric concepts and properties. This similarity can be explained by the fact that all three participants were selected because they demonstrated high ability in their respective spatial component and high performance in solving the 3D geometry problems. Thus, differences in spatial ability were more apparent in representing, manipulating, and constructing spatial information than in applying established geometric concepts.

This finding is important for mathematics teacher education. It suggests that strong geometric conceptual knowledge does not necessarily eliminate difficulties in spatial representation. A pre-service teacher may understand the mathematical property underlying a problem while still experiencing difficulty representing the corresponding 3D configuration. Consequently, preparation of future mathematics teachers should address both **geometric conceptual knowledge and spatial reasoning processes**.

6. Interaction among the Four Types of 3D Geometric Reasoning

One of the most important findings of this study is that the four types of reasoning do not occur independently. Rather, they **interact and may occur simultaneously depending on the demands of the task**.

For example, when determining the edge length of a cube from the number of unit cubes, **spatial structuring** is required to understand how the unit cubes form the larger solid, while **measurement reasoning** is needed to relate the number of cubes to the dimension being sought. Similarly, when drawing the front, top, and side views of a 3D object, **3D object manipulation** is required to represent the object from different viewpoints, while **conceptualization of mathematical properties** supports the application of geometric properties underlying those representations.

Conceptualization of mathematical properties appears to have a particularly integrative role. It frequently accompanies manipulation, spatial structuring, and measurement because mathematical properties provide the conceptual basis for interpreting and validating spatial relationships. Consequently, the four reasoning types should not be viewed as isolated categories but as **interdependent dimensions of 3D geometric reasoning**.

This finding has implications for the interpretation of the indicators developed in this study. The indicators should not be interpreted as representing completely separate cognitive processes. Instead, they provide an analytical framework for identifying the dominant characteristics of reasoning occurring during a particular problem-solving episode. One indicator may involve more than one reasoning type, but it can be classified according to the reasoning component that is most prominent in the participant's activity.

From this perspective, the findings extend the framework of Pittalis and Christou (2010) by illustrating how the four types of reasoning are manifested dynamically during the problem-solving process of mathematics pre-service teachers. The findings further suggest that spatial ability provides a cognitive resource through which these forms of reasoning are enacted, while geometric knowledge provides the conceptual basis for interpreting and validating the resulting representations and solutions.

7. Overall Discussion

Taken together, the findings indicate that **spatial ability does not simply determine whether a pre-service teacher can solve a 3D geometry problem; rather, it shapes how particular forms of reasoning are enacted during the solution process**. All three participants were capable of using the four reasoning types, but differences in spatial visualization, orientation, and relations became visible in the way they manipulated representations, organized spatial structures, and coordinated geometric quantities.

The findings therefore support a process-oriented interpretation of the relationship between spatial ability and 3D geometric reasoning. Spatial visualization was particularly evident in mental construction and transformation of 3D objects; spatial orientation

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was closely associated with interpreting objects from different viewpoints; and spatial relations supported the coordination of relationships among components of a geometric configuration. These components subsequently interacted with measurement and conceptualization of mathematical properties.

The findings also reinforce the importance of studying reasoning through learners' **actions, representations, explanations, and verification processes**, rather than relying only on final answers. The qualitative evidence reveals that two participants may arrive at a correct answer through somewhat different spatial processes, while a participant may demonstrate strong conceptual understanding but experience difficulty with a particular spatial representation.

For mathematics teacher education, this has an important implication. Developing pre-service teachers' competence in 3D geometry should not be limited to teaching formulas and geometric properties. Learning activities should provide opportunities to **mentally manipulate 3D objects, interpret multiple viewpoints, construct and deconstruct spatial structures, coordinate dimensions, and justify geometric relationships**. Such activities may strengthen the integration of spatial ability and mathematical reasoning required for effective teaching of 3D geometry.

IV. CONCLUSIONS

This study concludes that mathematics pre-service teachers with high spatial visualization, spatial orientation, and spatial relations abilities demonstrated all four types of reasoning in solving three-dimensional (3D) geometry problems: **3D object manipulation, spatial structuring, measurement, and conceptualization of mathematical properties**. These four types of reasoning were evident across the problem-solving stages of understanding the problem, planning and executing the solution, and checking the solution.

The three participants showed similar patterns at the understanding and checking stages. At the understanding stage, they were able to construct relationships among 3D objects, their representations, unit-cube arrangements, geometric measurements, and relevant mathematical properties. At the checking stage, they evaluated the consistency of representations, spatial structures, measurements, procedures, calculations, and the mathematical properties used. This indicates that 3D geometric reasoning involves an integrated process of constructing, organizing, measuring, conceptualizing, and verifying geometric information.

Differences among the three participants were more apparent during the planning and execution stages, particularly in tasks requiring spatial representation and transformation. The spatial visualization participant demonstrated strong ability to mentally manipulate 3D objects, construct alternative representations, organize unit-cube structures, determine dimensions, and coordinate these representations with geometric measurements. The spatial orientation participant demonstrated the four types of reasoning but experienced difficulty in representing the top view and in reconstructing certain 3D structures. Meanwhile, the spatial relations participant successfully established relationships among the components of 3D objects and generated multiple representations, although the resulting graphical representations were not always optimal.

These findings suggest that **spatial visualization, spatial orientation, and spatial relations play different roles in the manifestation of 3D geometric reasoning**. Spatial visualization was particularly evident in manipulating and transforming mental representations of 3D objects; spatial orientation was more closely related to interpreting objects from different viewpoints; and spatial relations supported the coordination of relationships among components of a geometric structure. However, these components did not operate independently. Rather, they interacted with the four types of reasoning according to the demands of the problem.

A further conclusion is that the four types of reasoning are **interrelated rather than discrete processes**. 3D object manipulation may occur together with spatial structuring when a participant transforms or reconstructs an object; measurement may depend on the spatial structure that has been constructed; and conceptualization of mathematical properties provides a basis for interpreting, calculating, and validating the geometric relationships involved. Thus, 3D geometric reasoning should be understood as a dynamic integration of spatial and mathematical processes rather than as a sequence of isolated reasoning skills.

The findings have implications for mathematics teacher education. Preparing prospective mathematics teachers to teach 3D geometry should not focus exclusively on formulas, procedures, and geometric properties. Learning experiences should also provide opportunities to manipulate and transform 3D representations, interpret objects from different viewpoints, construct and deconstruct spatial structures, relate unit cubes to dimensions and volume, and justify the use of geometric properties. Such experiences may strengthen the integration of spatial abilities and mathematical reasoning needed for effective teaching and learning of 3D geometry. Overall, this study contributes a qualitative characterization of how the three components of spatial ability are manifested in the reasoning of mathematics pre-service teachers when solving 3D geometry problems. The findings extend the understanding of 3D geometric reasoning by showing that **the nature of reasoning is reflected not only in the correctness of the final solution but also in how pre-service teachers represent, transform, structure, measure, conceptualize, and verify geometric information throughout the problem-solving process**.

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